NUMERICAL CALCULATION OF THE MAGNETIC CUMULATION PROCESS

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A numerical calculation is made of the process in the MK-1 magnetocumulative generator under the assumption that the magnetic flux is constant and the tube contraction process is one-dimensional. The instantaneous detonation scheme is adopted. The effect of initial magnetic field intensity and relative size of the cavity on the magnitude of the maximal magnetic field intensity obtained inside the tube is studied.

Methods for creating ultrastrong magnetic fields with the aid of an explosion have been described in [1-3]. In the following we examine the numerical calculation of the process in the MK-1 magnetocumulative generator. In this generator there is explosive, symmetric contraction of a metal tube with an internal axial magnetic field having the initial intensity H_0. If the tube walls are perfectly conducting its magnetic flux Φ remains constant in the compression process

$$\Phi = \pi a^2 H = \pi a_0^2 H_0 = \text{const}$$

Here a_0 and a are the initial and instantaneous internal radii of the tube. We denote the initial and instantaneous outside tube radii respectively by b_0 and b. The initial tube wall thickness is $\Delta_0 = b_0 - a_0$.

The magnetic field intensity H and its pressure p_m on the tube walls follow the law

$$H = H_0 a_0^2 / a^2, \qquad p_m = p_{m0} a_0^4 / a^4$$

Here p_{m0} is the initial magnetic field pressure.

The magnetic field intensity and its pressure are connected by the relation

$$p_m = H^2 / 8\pi$$

The magnetic cumulation problem in the general case is a two-dimensional (axisymmetric) unsteady gasdynamic problem. In the case of simultaneous contraction of the shell by a converging detonation wave, the problem with adequate approximation may be considered one-dimensional in the entire region which is not affected by the end-face rarefaction waves.

In the present paper we examine the shell motion under the action of initially stationary detonation products having the isentropic exponent k = 3 and initial parameters corresponding to an ideal (instantaneous) detonation [4]

$$p_0 = {}^1/_8 \rho_n D^2, \qquad \rho_0 = \rho_n, \qquad c_0 = ({}^3/_8)^{1/2} D \simeq 0.61 D$$

Here p_0, ρ_0, c_0 are respectively the initial pressure, density, and sound speed in the detonation products, ρ_n is the explosive density, D is the detonation velocity.

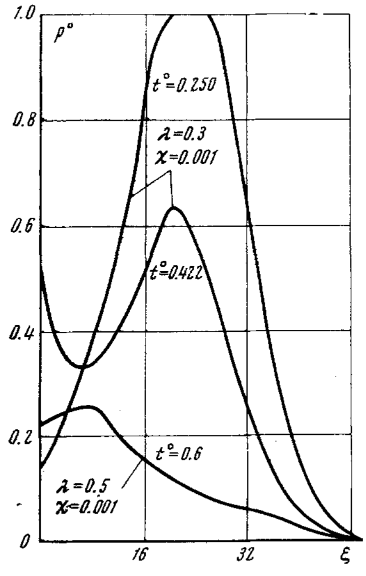

Fig. 1

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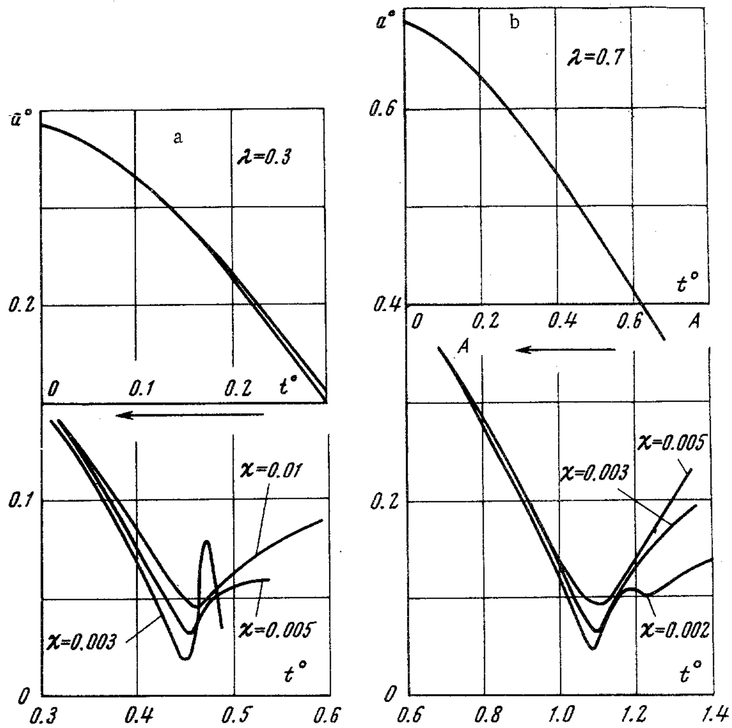

Fig. 2

For generality we examine the problem with an external thin shell around the charge. The material of both shells is assumed incompressible and having no strength. Reduction of the magnetic flux magnitude owing to finite conductivity of the tube is neglected. The motion of the detonation products up till the moment the inner shell stops is isentropic. The system of equations has the form

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho v)}{\partial r}+\frac{\rho v}{r}=0, \qquad \frac{\partial v}{\partial t}+v\frac{\partial v}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}=0, \qquad p=p_0\rho^3/\rho_0^3 \tag{1}$$

with the initial conditions

$$b_0\leqslant r\leqslant R_0, \qquad p=p_0, \qquad \rho=\rho_0, \qquad c=c_0, \qquad v=0$$

Here b_0, R_0 are respectively the inner and outer radii of the charge. The boundary conditions are:

at the outer shell

$$M_1 dv\,/\,dt=2\pi Rp$$

R is the instantaneous inner radius of the outer shell;

at the inner shell

$$M_2 dv\,/\,dt=2\pi\,(ap_m-bp)$$

Here M_1, M_2 are the masses of the outer and inner shells (per unit length). The values of the radii a and b are connected by the relation which follows from the shell incompressibility condition

$$b^2-a^2=b_0^2-a_0^2$$

The problem solution depends on the dimensionless parameters

$$\delta=\Delta_0\,/\,b_0, \quad \lambda=b_0\,/\,R_0,$$
$$\omega=\rho_t\,/\,\rho_n, \quad \beta=m\,/\,M_1,$$
$$\chi=p_{m0}\,/\,p_0$$

Here ρ_t is the tube material density, m is the explosive mass per unit charge length.

In place of the parameter δ we can use the specific mass of the explosive per unit mass of the inner shell $\gamma=m/M_2$. Obviously, γ is connected with δ and λ by the relation

TABLE 1

λ	γ	χ = 0.002		χ = 0.003		χ = 0.005		χ = 0.010	
		t^*	p_m^*	t^*	p_m^*	t^*	p_m^*	t^*	p_m^*
0.3	50	—	—	0.452	235.5	0.460	58.54	0.462	17.44
0.5	15	—	—	0.750	196.0	0.752	46.60	—	—
0.7	5.2	1.08	87.3	1.10	34.66	1.11	13.04	—	—

TABLE 2

λ	R_0 mm	H_0 = 71 000 Oe			H_0 = 87 000 Oe			H_0 = 112 000 Oe			H_0 = 159 000 Oe		
		t^*	p_m^*	H^*	t^*	p_m^*	H^*	t^*	p_m^*	H^*	t^*	p_m^*	H^*
0.3	166.7	—	—	—	17.65	23.55	24.3	18.35	5.85	12.15	18.0	1.74	6.63
0.5	100	—	—	—	17.6	19.6	22.2	17.6	4.66	10.85	—	—	—
0.7	71.4	18.1	8.73	14.83	18.4	3.47	9.35	18.6	1.30	5.74	—	—	—

$$\gamma = \frac{1}{\omega}\frac{1-\lambda^2}{\lambda^2}\frac{1}{\delta(2-\delta)}$$

The problem is examined in the dimensionless variables

$$r^\circ = r / R_0, \quad t^\circ = c_0 t / R_0,$$
$$v^\circ = v / c_0, \quad p^\circ = p / p_0,$$
$$\rho^\circ = \rho / \rho_0$$

Integration of the quasilinear system (1) was made by a finite difference method on a BÉSM-3M digital computer at the Institute of Problems of Mechanics of the Academy of Sciences of the USSR under the direction of L. A. Chudov. In all cases the cumulative shell thickness Δ_0 was taken to be 0.02 times the radius of the inner charge cavity b_0 (δ = 1/50). The cavity relative dimension $\lambda = b_0/R_0$ was 0.3, 0.5, 0.7, the exponent ω = 5. The relative initial magnetic field pressure χ was 0.001, 0.002, 0.003, 0.005, and 0.01. The pressure distribution in the detonation products as a function of the dimensionless coordinate $\xi = (r-b)/(R-b)$ for different times is shown in Fig. 1; the motion of the inner cavity with the magnetic field is shown in Fig. 2.

For large cavities and thick walls, as a result of the small thickness of the outer shell ($\Delta_R = {}^1/_5\,\Delta$) the distribution approaches that for unilateral discharge. The pressure peak in the gas occurs near the cumulative shell. For a thin cumulative shell (casing) the process is close to bilateral discharge.

In the initial stage of the motion the field has practically no effect. In the final stage of the motion the rapid increase of the magnetic field pressure leads to intense retardation of the shell and it comes to a stop. In this case the retardation of the gas flow approaching the thin shell leads to a sharp gas pressure rise and formation of a reflected shock wave. The subsequent motion of the gas is adiabatic but not isentropic.

Table 1 shows the dimensionless times until the cavity stops and the dimensionless pressure p_m^* at the moment of stopping. The relative intensities of the magnetic field at the moment of stopping are defined by the relation

$$H^* / H_0 = (p_m^* / \chi)^{1/2}$$

As we would expect, the pressure and field intensity at the moment of stopping decrease with increase of the initial field pressure. For example, for λ = 0.3 increase of χ from 0.003 to 0.005 leads to four-fold reduction of p_m^*. Increase of λ for a fixed value of χ leads to a reduction of p_m^*. The dimensionless time t^* when the cavity stops in the considered χ range depends very little on χ and is determined basically by the relative cavity dimension λ (Fig. 3).

The relative cavity contraction a^*/a_0 at the moment of stopping varies 0.06 (λ = 0.3, χ = 0.003) to 0.14 (λ = 0.7, χ = 0.005). The relative outer shell radius at the moment of stopping is obviously defined by the relation

$$b^* / a_0 = (b_0^2 / a_0^2 + a^{*2} / a_0^2 - 1)^{1/2}$$

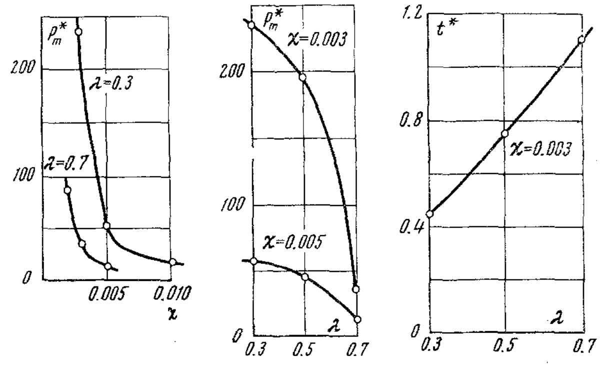

Fig. 3

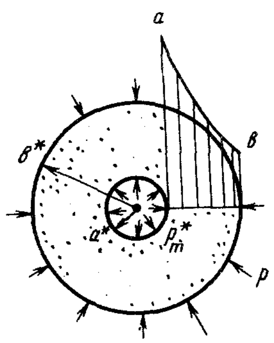

Fig. 4

For $a^* = a_0 = 0.06$ the quantity b^*/a_0 amounts to 0.209. The shell cross section at this moment is shown in Fig. 4. It is obvious that for the motion near the axis the boundary condition of the form

$$M_2 dv/dt = 2\pi (ap_m - bp)$$

previously presented for the thin shell is satisfied only approximately because of variability of the velocity through the shell thickness. From the equation of continuity for the incompressible shell we have

$$v = \dot{a}a/r = \dot{b}b/r, \qquad \dot{a}/\dot{b} = b/a$$

Here $\dot{a} = da/dt$, $\dot{b} = db/dt$ are the velocities of the cavity and the outer shell surface. In the present case the velocity ratio $\dot{a}/\dot{b}$ is 3.5.

At a definite time the shell stops and then its expansion reverses under the action of the magnetic field pressure. It is important to note that stopping occurs not at the time "...when the magnetic pressure balances the explosion pressure," as erroneously assumed in [3], but rather somewhat later. The magnetic field pressure on the shell at this moment is much greater than the external pressure of the detonation products. For example, for $\lambda = 0.5$, $\chi = 0.003$, $p_m^* = 196$, $p_b^* = 8.79$. With account for the inner and outer surface areas the forces on the shell are in the ratio 6.7:1.

Shell pulsations may occur in the expansion process as a result of interaction of the thin shell with the reflected shock wave in the detonation products. The most marked pulsations arise from small λ and χ (Fig. 2).

Table 2 shows the dimensional values of the parameters t^* (μ sec), p_m^* (Mbar), H^* (MOe) at the moment of stopping for the shell having the outer diameter $2R_0 = 100$ mm. The initial shell thickness was $\Delta_0 = 1$ mm. The ideal detonation pressure was taken as $p_0 = 100$ kbar ($D = 7000$ m/sec, $c_0 = 4270$ m/sec). The initial magnetic field pressure is 200, 300, 500, and 1000 bar (the corresponding magnetic fields are $71 \cdot 10^3$, $87 \cdot 10^3$, $112 \cdot 10^3$, and $159 \cdot 10^3$ G).

The maximal magnetic field intensity in the considered example is $24.3 \cdot 10^6$ Oe (field pressure 23.55 Mbar). It is interesting to note that the time from initiation of the motion until stopping of the shell for all values of λ and χ changes very slightly (from 17.6 to 18.6 m/sec). The average cavity velocity $\langle v \rangle = (a_0 - a^*)/t^*$ is 2.5 km/sec.

In comparison with the theoretical and experimental values for a cumulative shell of approximately the same dimensions ($2R_0$ = 3-4 inches, initial magnetic field 50-150 kG), presented in [3] ($\langle v \rangle = 4$ km/sec, $t^* \approx 10$ μsec), this calculation yields slower collapse. This is to be expected, since the adopted scheme with initially stationary gas, corresponding to instantaneous detonation, leads to lower values of the initial shell accelerations than the values obtained in the experiments using a converging detonation wave.

Within the limits of the scheme with instantaneous detonation, we can account for the effects owing to convergence of the detonation wave and its reflection from the shell, introducing the corresponding increase of the stationary gas parameters. For example, taking as the initial gas parameters the characteristics of the detonation products at the moment the detonation wave reflects from the shell ($p_0 = 2.37 p_{C-J}, p_{C-J} =$

$^1/_4\rho_n D^2$, $c_0 = D$), we obtain for the same values of χ the time for the shell to come to a stop $t^* = 10.7$–11.3 μsec, $\langle v \rangle = 4.1$ km/sec, which is in good agreement with the data of [3].

We note that for the considered magnetic field pressures it is theoretically necessary to account for the compressibility of the shell material. Account for the compressibility may lead to some reduction of the limiting field intensity.

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